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## UNIT 1: EXPRESSIONS, EQUATIONS, AND FUNCTIONS

### 1-1 The Real Numbers

***Mathematics*** is the study of numbers, shapes, arrangements, relationships, and reasoning.

A ***number*** is an idea; we represent numbers by using ***symbols***, called **numerals or numerical expressions.**

***Counting Numbers*** or ***Natural Numbers*** are represented by the symbols
**1, 2, 3, 4, 5, 6, 7, 8, 9, 10, …**

Each counting number has a ***successor*** that is 1 more than that number. For example, the successor of 1 is 2, the successor of 2 is 3, and so on.

The smallest counting number is 1, and there is NO largest counting number. Zero is NOT a counting number.

***Whole Numbers*** are represented by the symbols **0, 1, 2, 3, 4, 5, 6, 7, 8, 9, …**

The smallest whole number is 0, and there is NO largest whole number.

A ***set*** is **a collection of distinct objects or elements.**
A set is usually indicated by enclosing its elements within a set of braces, { }. ← curly brackets

A ***finite set*** is a set whose elements **can be counted**.
Example: The set of digits: {0, 1, 2, 3, 4, 5, 6, 7, 8, 9}

An ***infinite set*** is a set whose elements **cannot be counted** because **there is no end to the set**.
Example: The counting numbers and the whole numbers are infinite sets.

This is in this set

The ***empty set*** or the ***null set*** is a set that **has no elements**.
The empty set is written as { } or ∅.
Example: The set of negative counting numbers is empty.

don't do this → {∅} not empty

An ***operation*** is a **procedure or a rule that tells you how to combine elements or numbers in a set to obtain other numbers.**

| OPERATION | SYMBOL | RESULT |
|---|---|---|
| Addition | + | Sum |
| Subtraction | - | Difference |
| Multiplication | × or • | Product |
| Division | ÷ or fraction | Quotient |

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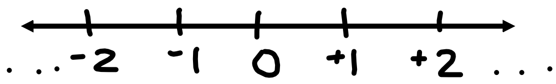
A ***numerical expression*** **is a way of writing a number in symbols.**
Example: $6 + 2 = 2 \times 2 \times 2 = 4 \times 2 = 1 \times 7 + 1 = 18 - 10 = 640 \div 80 = 8$

**The Integers**

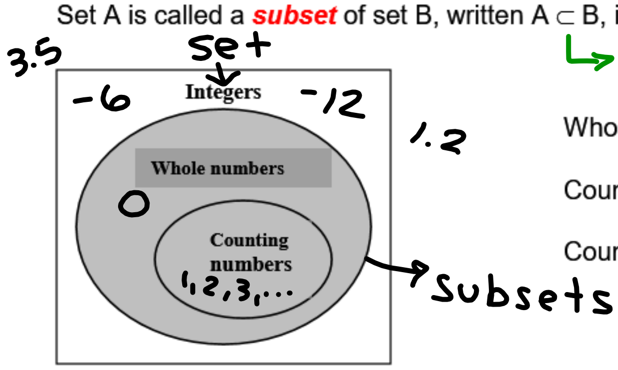



A ***positive number*** is **a counting number.**
A ***negative number*** is the ***opposite*** of a positive number.
Zero is **neither** positive nor negative.

The ***set of integers*** consists of the **positive whole numbers, $\underbrace{\text{the negative whole numbers}}$, and zero.**
→ their opposites

The set of integers can be shown as {…, -3, -2, -1, 0, +1, +2, +3, …} or as
{0, +1, -1, +2, -2, +3, -3, …}.

Set A is called a ***subset*** of set B, written A ⊂ B, if every element of set A is also an element of set B.




↳ "is a subset of"

Whole numbers ⊂ Integers

Counting numbers ⊂ Integers

Counting numbers ⊂ Whole numbers

***Inequalities*** occur when one number is not necessarily equal to another number. We use the following inequality symbols to show inequalities:

| | | | | | |
|---|---|---|---|---|---|
| $3 < 5$ | less than | $<$ | greater than | $>$ | $5 > 3$ |
| $3 \leq 5$<br>$5 \leq 5$ | less than or equal to | $\leq$ | greater than or equal to | $\geq$ | $5 \geq 3$<br>$3 \geq 3$ |
| | not equal | $\neq$ | | | |

$5 \neq 3$
$3 \neq 5$

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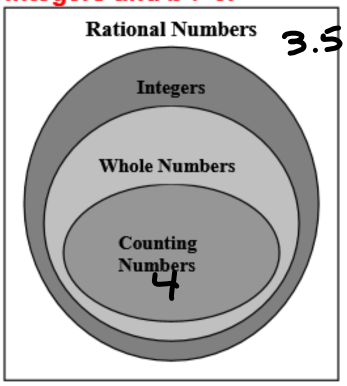
**The Rational Numbers** $\frac{a}{b}$ represents a fraction/decimal.

***Rational numbers*** are **all numbers that can be expressed in the form $\frac{a}{b}$ where *a* and *b* are integers and $b \neq 0$.**

$\frac{a}{0} \rightarrow$ division by 0 is UNDEFINED.




The counting numbers, whole numbers, and integers are all subsets of the rational numbers.

$\frac{8}{2} = 4$ $\frac{7}{2} = 3.5$

**Properties of Rational Numbers**

- The set of rational numbers is everywhere dense. Given two unequal rational numbers, it is always possible to find a rational number between them.

To find a rational number between two others, you can find the mean (average) of the two numbers.

- Every rational number can be expressed as either a terminating decimal or a repeating decimal.

(ends)

Terminating: $\frac{1}{2} = 2\overline{)1.0000}$ with quotient $0.5000$ $= .5$ Repeating: $\frac{1}{3} = 3\overline{)1.00000000}$ with quotient $0.333333....$

- Every rational number can be expressed as a repeating decimal.

Express a decimal as a Rational Number: $\rightarrow$ write as a fraction

1. Read it using place value: 0.8 **= 8 tenths.**

Place Value:

2. Write it as a fraction: 0.8 **=** $\frac{8}{10}$

3. Reduce it, if possible: $\frac{8 \div 2}{10 \div 2} = \frac{4}{5}$

. . . Hundreds Tens Ones . Tenths Hundredths Thousandths . . .

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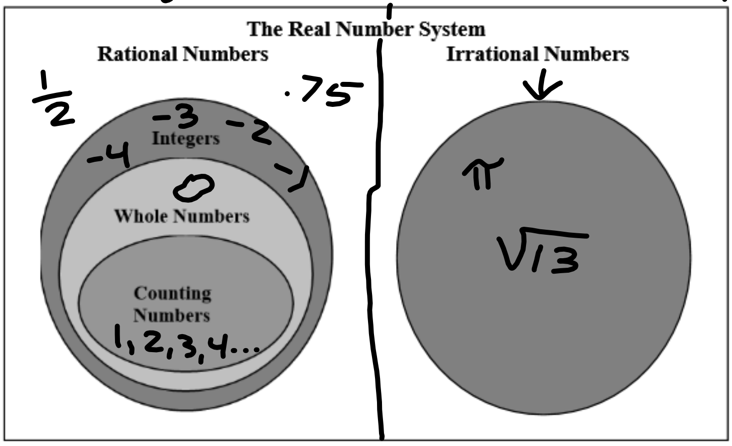
**The Irrational Numbers** → Not Rational

***Irrational Numbers*** are **nonrepeating, and nonterminating decimals; also, the square root of a nonperfect square number.**

Perfect square = $\sqrt{16} = 4$ (rational)

non-perfect square: $\sqrt{13}$ (irrational)




The irrational numbers are a subset of the real number system. They are completely separate from the subset of rational numbers. A number can be rational OR irrational, but not both.

***Irrational Numbers*** are ALWAYS **nonrepeating and nonterminating (must be BOTH).**

Examples: $\pi$ = **3.141592654 …** **(only ends because the calculator screen ends)**

$\sqrt{2}$ = **1.414213562 …** **(only ends because the calculator screen ends)**

$\frac{\pi}{3}$ = **1.047197551 …** **(only ends because the calculator screen ends)**

Regents-type questions:

So, what happens if you add two *rational numbers* together? Or if you subtract two *rational numbers?* Or if you multiply two *rational numbers?* Let's try it….

Integers:
$2 + 7 = 9$ rational or irrational?
$2 - 7 = -5$ rational or irrational?
$2 \times 7 = 14$ rational or irrational?

Fractions:
$\frac{7}{10} + \frac{4}{5} = \frac{15}{10}$ rational or irrational?
$\frac{7}{10} - \frac{4}{5} = \frac{-1}{10}$ rational or irrational?
$\frac{7}{10} \times \frac{4}{5} = \frac{28}{50}$ rational or irrational?

$\frac{7}{10} + \frac{4 \cdot 2}{5 \cdot 2}$

$\frac{7}{10} + \frac{8}{10} = \frac{15}{10}$

$\frac{7}{10} - \frac{4}{5}$

$\frac{7}{10} - \frac{8}{10} = \frac{-1}{10}$

$\frac{7}{10} \times \frac{4}{5} = \frac{28}{50}$

→ across

Integer/fraction: $8+\frac{2}{3}=$ $\frac{26}{3}$ rational or irrational?

$8-\frac{2}{3}=$ $\frac{22}{3}$ rational or irrational?

$8\times\frac{2}{3}$ $\frac{16}{3}$ rational or irrational?

$\frac{3\cdot 8}{3\cdot 1}+\frac{2}{3}=\frac{24}{3}+\frac{2}{3}=\frac{26}{3}$

$\frac{24}{3}-\frac{2}{3}=\frac{22}{3}$

$\frac{8}{1}\times\frac{2}{3}=\frac{16}{3}$

So, it seems that we can safely say that the sum, difference, or product of two rational numbers is always Rational.

Now, what if we do these operations on *irrational numbers?* Or with *rational/irrational numbers?*

$\sqrt{2}+\sqrt{8}=$ _____ rational or irrational? 4.242640687...

$\sqrt{2}-\sqrt{8}=$ _____ rational or irrational? −1.414213562...

$\sqrt{2}\times\sqrt{8}=$ _____ rational or irrational? $=\sqrt{2\cdot 8}=\sqrt{16}=4.0$

$\sqrt{3}\times\sqrt{5}=$ _____ rational or irrational? $=\sqrt{3\cdot 5}=\sqrt{15}$

So, it seems that the sum or difference of two irrational numbers will be Irrational, and the product of two irrational numbers could be Rational or Irrational, depending on what the numbers are.

Examples:

1.

Which statement is *not* always true?

A) The product of two irrational numbers is irrational. could be rational

B) The product of two rational numbers is rational. true

C) The sum of two rational numbers is rational. true

D) The sum of a rational number and an irrational number is irrational. true

2.

For which value of *P* and *W* is *P* + *W* a rational number?

A) $P=\frac{1}{\sqrt{4}}$ and $W=\frac{1}{\sqrt{9}}$ R R R+R

B) $P=\frac{1}{\sqrt{3}}$ and $W=\frac{1}{\sqrt{6}}$ I I I+I

C) $P=\frac{1}{\sqrt{6}}$ and $W=\frac{1}{\sqrt{10}}$ I I I+I

D) $P=\frac{1}{\sqrt{25}}$ and $W=\frac{1}{\sqrt{2}}$ R + I

P and W must BOTH be rational numbers

square root of perfect square = R

square root of non-perfect square = I